Can Bilateral Trade Explain Business Cycle Correlations?

Xiao Wang*

October 30, 2009

Abstract

OECD data indicate that output levels are more highly correlated among countries with higher bilateral trade intensity. Bilateral trade builds up a bridge between countries such that a positive productivity shock in the home country makes consumers richer to purchase more imported goods, and accelerates production in the foreign country. But the complementarity effect in a standard real business cycle model is not strong enough to replicate data results. This paper adds input-output matrix into a standard business cycle model, and the new model generates stronger complementarity effect. A positive productivity shock in one sector can spread to the whole economy by requiring intermediate goods from other sectors, and other sectors require more inputs in further rounds. Multiplier effects from the interaction in the input-output table enlarge the complementarity effect. Simulation results from the new model are consistent with data estimation.

JEL classification codes: E3 F1 F4 O1

Keywords: Trade intensity, International Real Business Cycles, Input-Output Matrix

1 Introduction

World trade has experienced significant growth in the last forty years. Exports and imports of goods and services in the world have doubled: the sum of exports and imports accounted for 26 % of world GDP in 1970 and 54 % of world GDP in 2005[1]. With the development of world

*Department of Economics, University of Wisconsin-Madison. xwang24@wisc.edu.

[1] World development indicator data base, World Bank.

trade, countries have built up stronger relationship. Figure 1 depicts countries with more bilateral trade tend to have more closely synchronized business cycles. [2] There are 378 observations on the graph, each point representing one bilateral pair among 28 countries: Australia, Austria, Belgium, Canada, Czech, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, U.K., U.S.[3]. The fitted least square line shows countries with more bilateral trade tend to have higher output correlations.

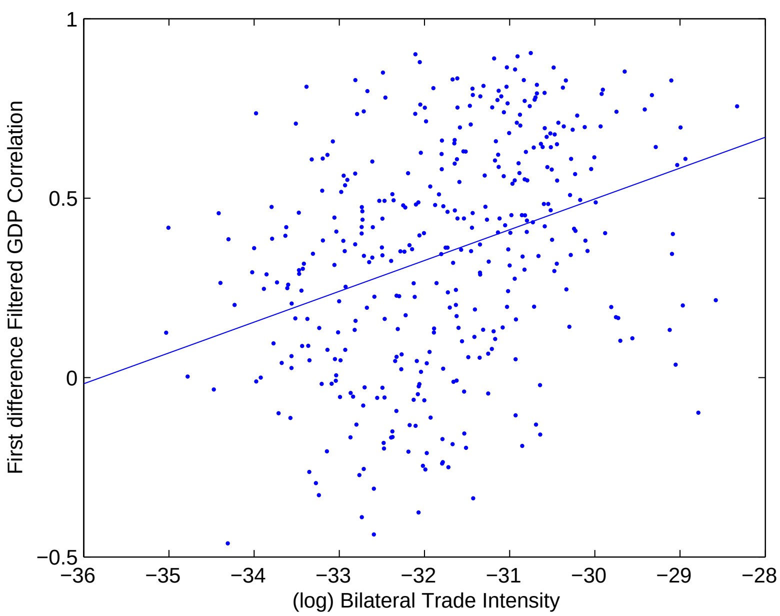

Figure 1: The relationship between bilateral trade and GDP co-movement

Data resource:OECD economic outlook and STAN.

Standard economic models can not fully explain the mechanism why countries with more bilateral trade tend to have synchronized output correlations. In Kose and Yi ([17], 2006), bilateral trade builds up a bridge between countries such that a positive productivity shock in the home country make consumers richer to purchase more goods shipped from the foreign country. Then increasing demand in the home country accelerates production in the foreign country and result in synchronized output correlation across countries. But on the other hand,

[2]This empirical regularity is first proposed by Frankel and Rose (1998, [13]) and then confirmed by other studies, such as Baxter and Kouparitsas (2005, [6]) for developed countries, Calderon, Chong, and Stein (2007, [9]) for developing countries.

[3]Details of the data set can be found in the next section.

foreign consumers find prices of imported goods are so attractive that they tend to consumer more imported goods, even if home and foreign goods are not perfectly substitutable. This effect tends to reduce the output correlation across countries. Thus standard models predict lower slope of bilateral trade on output correlations, compared with data estimation.

This paper adds a new feature into a standard two-country real business cycle: the use of intermediate goods in production. The input-output structure may generate stronger complementarity effect than the standard model. A simple example is quite helpful for understanding. Assume that there is a positive productivity shock on the computer industry in the home country. In the standard model without intermediate goods, home consumers would purchase more goods, both domestic and abroad, as their income has increased. But the expanding demand in the home country diminishes quickly after the shock. In a model featuring with intermediate goods, the complementarity effect can spread to the whole economy. First of all, the computer industry needs intermediate goods from abroad, for example, plastic, which increases the foreign output in the first round. Second, The use of computer, such as replacing human manipulation with computer-controlling system, improves production in other industries in the home country, which sets off another round of demand for foreign goods. The infinite rounds of multiplier effect is summarized by the input-output table. At the same time, the complementarity effect also works through the channel of consumers, as in the standard model.

Intensive use of intermediate goods in production motivates this paper to model it into the standard model. The value of intermediate goods accounts for 47% of gross production in U.S.[4]. In twenty major OECD countries, intermediate goods account for average 45.2% of total imports[5]. I model the production process with intermediate inputs through the input-out matrix, not through multiple stages of production. There are three reasons why I choose the "roundabout" rather than "in-line" production. First of all, "roundabout" production can generate multiplier effect, which is the key mechanism in this paper, but "in-line" production can not. In "in-line" production, goods produced in downstream sectors can not be used to produce upstream goods. Thus a productivity shock on the downstream sector stops to spread in the production sector after the first round. Second, it is convenient to study interactions across sectors through input-output matrix. Elements in the input-output matrix identify how much input needed to produces one unit value of certain product. Any change in a specific sector would affect other sectors if goods in that sector is used as inputs for other sectors. "In-

[4] OECD STAN database.
[5] Table 1 in Ramanarayanan(2009, [18]).

line" production, in contrast, is hard to capture the effect of downstream sectors on upstream sectors. Third, as pointed out by Basu (1995, [5]), even though irreversible "in-line" production seems plausible, it is difficult to determine which sector is the initial sector in the real world data. Empirically large elements in the input-output matrix are generally the diagonal entries. In other words, goods from its own sector account for a large part of intermediate inputs in a certain sector. "In-line" production can not explain this case.

The use of input-output matrix does not necessarily imply that firms use intermediate goods by Leontief technology, or the elasticity of substitution among inputs is zero. I assume a positive constant elasticity of substitution among inputs. Input-output matrix measures the cost share ratio of different inputs at the steady state.

To understand the new characteristics in this paper, I compare this paper with four sets of papers.

1. Kose and Yi (2001, [16]; 2006, [17]) develop a standard real business cycle model to examine output co-movements under different trade intensities. Countries in their model specialize in producing one good and consumers in each country consume the combination of goods, both domestic goods and imports. They find the increased correlations with higher trade intensity falls far short of the empirical findings.

This paper introduces input-output matrix into the production. There are multiple sectors in each country and the country produces goods in all sectors. Goods can be used as consumption or intermediate goods. Goods from two countries in the same sector are substitutable with certain elasticity of substitution. Firms in one sector needs inputs from all sectors, according to the input-output matrix. By modeling the complementarity of inputs, the model explores a new channel transmitting shocks across countries and regression based on model simulation is consistent with the empirical finding.

2. Burstein, Kurz, and Tesar (2008, [8]) explicitly model the role played by intermediate goods when considering the relationship between trade and output synchronization. They focus on production sharing, the trade in intermediate goods that is part of vertically integrated production networks cross international borders. In other words, the production structure in their model is "in-line". Like Kose and Yi (2006), the final goods sector can not directly affect the production of intermediate goods through input-output matrix. And they consider the output correlation in the traded-goods sector. As the volume of trade increases, the cross-country correlation between total output is actually negative in their model. This implication

contradicts data evidence.

This paper explores the interactions among sectors, which is essential to explain the relationship between bilateral trade and output synchronization. As shown in later sections, I compare two models: an original Burstein-Kurz-Tesar model and a Burstein-Kurz-Tesar model nested with input-output structure. The second model generates simulation results consistent with data, while the first model does not.

3. di Giovanni and Levchenko (2008, [14]) control common shocks and confirm the existence of relationship between trade and output correlation. They argue that common shocks may happen to be stronger for countries that have high levels of bilateral trade. To sweep out the concern about omitted common shocks, they add country-pair and industry pair fixed effect into regression. They furthermore empirically demonstrate that vertical linkages in production are important in understanding the relationship between bilateral trade and business cycle synchronization. They use input-output matrix to measure how a sector in one country can affect the production in the other country. But their estimates are limited in the manufactured good, not the aggregate output. In addition, their paper only estimates reduced-form equations, and can not be suited toward answering the question through which channel the bilateral trade can improve the output co-movement across countries.

This paper explicitly builds up a two-country real business cycle model and thus can examine the relationship between bilateral trade and output correlation in the framework of general equilibrium. Furthermore, this paper incorporates all sectors in the economy, not limited in the manufactured goods.

4. Ambler, Cardia, and Zimmermann (2002, [1]) construct a two-country real business cycle model and incorporates the input-output structure. But the input-output matrix is not estimated from the real data, which is necessary to obtain reasonable results. They do not examine the implications of their model on the relationship between trade and output correlation, which is the focus of this paper.

The rest of this paper is organized as follows. Section 2 displays empirical evidence on how bilateral trade may explain output synchronization. Section 3 presents the benchmark model. Section 4 calibrate the benchmark model and compare the quantitative result with other models. Section 5 concludes.

2 Empirical Evidence

In this section, I use an updated data set to verify the empirical regularity that countries with more bilateral trade exhibit higher business cycle correlations. I run regressions between bilateral output correlation and trade intensity, using OLS and instrument estimation. I find coefficients are positive and statistically significant, robust to controlling Total Factor Productivity (TFP) correlations or not.

I use a sample of 28 OECD countries[6]. There are 378 observations for bilateral pairs of 28 countries. This is a sample of annual data running from 1988 to 2004 for all countries[7].

I measure the bilateral trade intensity as the sum of imports within a country pair divided by the product of outputs

$$trade_{ijt} = \frac{m_{ijt}}{(y_{it} * y_{jt})} + \frac{m_{jit}}{(y_{it} * y_{jt})}, \qquad i, j = 1, \cdots, N,$$

where m refers to the imports of one country from the other country, y denotes the aggregate output, and subscripts i, j denote countries, t time. It is consistent with gravity models to use the product, not the sum of aggregate outputs as in Frankel and Rose (1998, [13]) to normalize the trade intensity[8]. The average trade intensity is simply

$$trade_{ij} = \sum_{t=1}^{T} trade_{ijt}.$$

I estimate two equations. The first equation updates the result in Frankel and Rose ([13], 1998)

$$corr(\hat{y}_i, \hat{y}_j) = \alpha + \beta * log(trade_{ij}) + \epsilon, \tag{1}$$

where $\hat{y}$ is logged and filtered GDP at constant price, $corr(\hat{y}_i, \hat{y}_j)$ is the correlation of de-trended outputs between two counties. Frankel and Rose (1998, [13]) argue that the correlation of business cycles across countries may depend on trade integration, and integration is also affected by policy. In other words, countries may synchronize their outputs because of the common policy,

[6] Australia, Austria, Belgium, Canada, Czech, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, U.K., U.S.

[7] Except Austria: 1995-2004; Czech: 1993-2004; Germany: 1992-2004; Hungary: 1992-2004; Korea: 1994-2004; Mexico: 1990-2004; New Zealand: 1990-2004; Poland: 1992-2004; Turkey: 1989-2004.

[8] see Deardorff(1998, [11]), Clark and van Wincoop([10], 2001).

Table 1: Can trade intensity explain business cycle synchronization? Estimation from data

Dependent variable:	output	correlation						
	(1)OLS		(2)IV		(3)OLS		(4)IV	
	HP	1std	HP	1std	HP	1std	HP	1std
log(trade intensity)	.10***	.09***	.11***	.11***	.05***	.04***	.07***	.06***
	(.02)	(.01)	(.02)	(.02)	(.01)	(.01)	(.02)	(.01)
TFP correlation					.82***	.75***	.81***	.74***
					(.04)	(.04)	(.04)	(.04)
R^2	.07	.11	.07	.11	.60	.57	.60	.57

*Note: regressand is bilateral correlation of real GDPs, Hodrick-Prescott filtered (HP) or first difference filtered (1std). Instrument for log (trade intensity): log(bilateral distance between capitals). Standard deviation recorded in parentheses. Intercepts not recorded. *** marks significance at 1% level. Sample size = 378. OECD economic outlook and STAN.*

which in the other hand drives up bilateral trade across countries. To avoid the endogeneity caused by the dependence, I estimate (1) by Instrument Variable (IV) method besides OLS. I use gravity variable as instruments of trade intensity: distance (mileage) between capital cities of two countries[9]. The OLS estimation ((1) in table 1) verifies the positive relationship between trade and output co-movement. IV regression ((2) in table 1) indicates that bilateral trade has positive effects on the correlation of business cycles, after controlling the endogeneity problem.

The second equation examines if trade intensity has additional explanatory power on output correlations, after controlling aggregate productivity shocks. The productivity z is computed from $log(z) = log(y) - .64 * log(l)$, where l is the total employment in one country[10]. Hence the correlation of de-trended productivity between country i and j is $corr(\hat{z}_i, \hat{z}_j)$, where $\hat{z}$ is the logged and de-trended productivity z. Now I am ready to estimate

$$corr(\hat{y}_i, \hat{y}_j) = \alpha + \beta_1 * log(trade_{ij}) + \beta_2 * corr(\hat{z}_i, \hat{z}_j) + \epsilon. \quad (2)$$

[9] I also try some other gravity variables: if two countries share the land border, and if two countries use a common language. The distance data comes from http://www.indo.com/cgi-bin/dist.

[10] I also compute $log(z)$ by $log(z) = log(y) - .64 * log(l) - .36 * log(k)$, wherer k is the total capital in one country. As the fluctuations of capital account for a small part of variance in total output, estimation results are robust to the alternative measure of Solow residual.

Both OLS and IV regressions ((3) and (4) in table **??**) confirm that trade intensity has additional explanatory power after controlling the correlation of total productivity shock.

3 Model

There are two ex-ante symmetric countries: home and foreign. Time is discrete and indexed by $t = 0, 1, 2, \cdots$. Each country has N sectors and produces N tradable goods, indexed by $h = 1, 2, \cdots, N$. Below I assume $N = 3$ for the convenience of describing the model. In every period, firms in each sector face a competitive market and maximize their profits subject to constant return-to-scale production technology using capital, labor, and intermediate goods.

Origins make goods different even in the same sector, for example, agricultural product from the home country and that from the foreign country are not perfect substitutes. Hence bilateral trade exists between two countries. Goods can be used as intermediate goods, consumption, and investment.

Households live forever in each country. They seek to maximize discounted sum of utility in infinite periods. Households earn income from wages and capital rents. Asset markets are complete such that there are state-contingent bonds hedging risks across countries.

As two countries are symmetric, I only specify the problem in the home country if possible. Analogous treatment can be done in the foreign country's problem.

3.1 Firms

A representative competitive firm in sector $h = 1, 2, 3$ in the home country produces goods from intermediate goods m_t^h(which will be explained in detail later), capital k_t^h, and labor l_t^h, according to Cobb-Douglas production function

$$y_t^h = z_t^h (l_t^h)^{\alpha_l} (k_t^h)^{\alpha_k} (m_t^h)^{\alpha_m}, \tag{3}$$

where z_t^h is the technology shock in sector h, $\alpha_l + \alpha_k + \alpha_m = 1$.

Technology shock is the source of uncertainty in this model. Define technology shock in all sectors as $z \equiv [z^1, z^2, z^3, z^{*1}, z^{*2}, z^{*3}]$. The technology shock z follows an AR(1) process

$$log(z_t) = Alog(z_{t-1}) + \epsilon_t, \tag{4}$$

where A is the persistence parameter, and ϵ_t is a normally distributed i.i.d. random vector with zero mean and variance-covariance matrix Σ. I denote the history of productivity shock up to period t as $s^t \equiv (s_0, s_1, \cdots, s_t)$ and all possible shocks in period $t+1$ as s_{t+1}. Therefore the innovation process equates the sum of all possible shocks, $\epsilon_t = s_t$. Every variable in the model is a function of productivity shock history s^t. For simplifying notations, I omit s^t in every variable and only specify future productivity shock s_{t+1} if necessary.

Firms sell product y_t^h to the home and foreign country d_t^h, d_t^{*h} at price $p_{d,t}^h, x_t p_{d,t}^{*h}$ respectively, where x_t is the real exchange rate. I assume iceberg cost arises in the international transportation. Only a fraction $1-\tau$ of goods reaches the foreign country:

$$d_t^h + \frac{d_t^{*h}}{1-\tau} = y_t^h. \tag{5}$$

In summary, a firm in sector $h (h = 1, 2, 3)$ in the home country chooses sales to the home and foreign country d_t^h, d_t^{*h}, capital k_t^h, labor l_t^h, intermediate goods m_t^h to maximize the discounted profit

$$E_0 \sum_{t=0}^{\infty} \Delta_t (p_{d,t}^h \cdot d_t^h + x_t p_{d,t}^{*h} d_t^{*h} - w_t l_t^h - r_t k_t^h - p_{m,t}^h m_t^h), \tag{6}$$

where Δ_t is the pricing kernel. As households own firms, the pricing kernel is the marginal rate of substitution of consumptions in period t and 0, which will be explained in detail later.

In the foreign country, firms in sector h face a similar problem as in the home country. They choose sales to the foreign and home country f_t^{*h}, f_t^h to maximize their profit.

3.2 Intermediate goods

Firms in sector h need to use goods within the sector and from other sectors as intermediate goods. As in equation (3), intermediate goods account for a fraction α_m in producing one unit value of good h. Input-output matrix Γ further defines how much inputs are needed in the expenditure on intermediate goods. Specifically, each element in column h of the input-output matrix $\Gamma_{sh}, s = 1, 2, 3$ tells the input value of good s; each element in row i $\Gamma_{ik}, k = 1, 2, 3$ tells how the good from sector i can be used by sector k. I normalize the input-output matrix such that the sum of elements in every column is one.

I assume all firms produce for both markets: manufactured inputs and final goods. Both domestic and foreign goods can serve either as intermediate inputs or final goods. In sector h, the composite goods of home and foreign products are

$$q_{t,s} = [\omega_s (d_t^s)^{\frac{\sigma_q - 1}{\sigma_q}} + (1-\omega_s)(f_t^s)^{\frac{\sigma_q-1}{\sigma_q}}]^{\frac{\sigma_q}{\sigma_q-1}}. \tag{7}$$

Firms in sector h uses intermediate inputs $q_{t,s}^h, s = 1, 2, 3$, where the subscript denotes where the input comes from, and the superscript denotes where the inputs can be used for. The elasticity of substitution across them is σ_m. All intermediate goods from difference sectors composite by the technology

$$m_t^h = [\sum_{s \in B_m^h} \Gamma_{sh} (q_{t,s}^h)^{\frac{(\sigma_m - 1)}{\sigma_m}}]^{\frac{\sigma_m}{\sigma_m - 1}}, \tag{8}$$

where B_m^h is the intermediate good set of h, as it is possible that producting good h uses some, not all goods from the economy[11].

3.3 Household

A representative household in the home country chooses consumption and investment to maximize its discounted sum of utilities, subject to its budget constraints. Consumption and investment are composite goods:

$$c_t + i_t = \prod_{h=1}^{3} (q_h^0)^{\theta_h}, \quad \sum_{h=1}^{3} \theta_h = 1, \tag{9}$$

where q_h^0 is the household's demand for composite goods in sector $h, h = 1, 2, 3$ as it consumes both domestic and imported goods; $\theta_h, h = 1, 2, 3$ are cost share of good h in expenditure of consumption and investment. I normalize the prices of consumption in each country as numeraire.

The household accumulates physical capital by a standard law of motion

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{10}$$

where δ is the constant depreication rate on $(0, 1]$.

With complete asset markets, the household spends income from wage, capital rent and

[11]When $\sigma_m = 1$, intermediate goods m^h is a Cobb-Douglas composite: $m_t^h = \prod_{s \in B_m^h} (q_{t,s}^h)^{\Gamma_{sh}}$.$\Gamma_{sh}$ directly measures the cost share of composite q_s^h in producing good h: $\frac{p_q^s q_s^h}{p_m^h m^h} = \Gamma_{sh}$. In the general case of CES function, Γ_{sh} measures steady-state cost share: $\frac{p_q^s q_s^h}{p_m^h m^h} (\frac{q_s^h}{m^h})^{\frac{1-\sigma_m}{\sigma_m}} = \Gamma_{sh}$.

maturing bonds b_t on purchasing consumption, investment and state-contingent bonds $b(s_{t+1}, s^t)$ in the next period:

$$c_t + i_t + \sum_{s_{t+1}} v(s^t, s_{t+1}) b(s_{t+1}, s^t) = b_t + w_t l_t + r_t k_t \tag{11}$$

where $l_t = \sum_{h=1}^{3} l_t^h, k_t = \sum_{h=1}^{3} k_t^h$, $v(s^t, s_{t+1})$ is the price of state-contingent bonds.

The problem of a representative household is

$$max \qquad E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\eta}(1-L_t)^{1-\eta})^{(1-\xi)} - 1}{1-\xi} \tag{12}$$

subject to the constrain (11).

Now it is ready to define the pricing kernel,the marginal rate of substitution of consumptions in period t and 0

$$\Delta_t = \beta \frac{U_c(c_t, 1-l_t)/p_t}{U_c(c_0, 1-l_0)/p_0}.$$

A foreign household faces a similar utility maximization problems with the budget constraint:

$$c_t^* + i_t^* + \sum_{s_{t+1}} \frac{x(s^t, s_{t+1})}{x_t} v(s^t, s_{t+1}) b^*(s_{t+1}, s^t) = b_t^* + w_t^* l_t^* + r_t^* k_t^*, \tag{13}$$

where $x(s^t, s_{t+1})$ is the real exchange rate in period $t+1$ at state s_{t+1}.

3.4 Equilibrium

Appendix A lists detailed first order conditions for the firm's and household's problem. The competitive equilibrium consists of prices $\{p_{d,t}^h\}, \{p_{d,t}^{*h}\}, \{p_{f,t}^h\}, \{p_{f,t}^{*h}\}, \{p_{q,t}^h\}, \{p_{q,t}^{*h}\}, \{p_{m,t}^h\}, \{p_{m,t}^{*h}\}$,

$\{w_t\}, \{w_t^*\}, \{r_t\}, \{r_t^*\}, \{x_t\}, \{v(s^t, s_{t+1})\}, h = 1, 2, 3$

and allocations $\{y_t^h\}, \{y_t^{*h}\}, \{d_t^h\}, \{d_t^{*h}\}, \{f_t^{*h}\}, \{f_t^h\}, \{c_t\}, \{c_t^*\}, \{i_t\}, \{i_t^*\}, \{k_t^h\}, \{k_t^{*h}\}, \{k_t\}, \{k_t^*\}$ $\{l_t^h\}, \{l_t^{*h}\}, \{l_t\}, \{l_t^*\}, \{q_{h,t}\}, \{q_{h,t}^*\}, \{q_{h,t}^j\}, \{q_{h,t}^{*j}\}, \{q_{h,t}^0\}, \{q_{h,t}^{*0}\}, \{m_t^h\}, \{m_t^{*h}\}, \{b_t\}, \{b_t^*\}, h, j = 1, 2, 3$

such that given prices, allocations solve the representative household's problem and firm's problem, and all markets clear.

4 Quantitative results

4.1 Parameterization

I assume two countries are symmetric and identify U.S. as the home country, a group of OECD countries as the foreign country[12]. Missing data in the use of intermediate goods restricts me to use some, not all of 27 countries to construct the counterpart of home country. I use US data to calibrate parameters except the productivity shock process, which involves both countries. Three sectors in the model are calibrated as "three big": (1) agriculture + mining + utility + construction; (2) manufacture; (3) service (wholesale and retail trade; restaurants and hotels; transport, storage,and communication; finance, insurance, real estate and business services; community social and personal services). I summarize all parameter values in table 2. Details are discussed below.

Table 2: Calibration

Parameter	Description	Value
α_m	Cost share of intermediate goods	.50
α_k	Cost share of capital	.18
α_l	Cost share of labor	.32
δ	Depreciation rate	.1
Γ	input-output matrix	$\begin{pmatrix} .16 & .05 & .01 \\ .43 & .74 & .08 \\ .41 & .21 & .91 \end{pmatrix}$
σ_m	Elasticity of substitution between different sectors	.5
σ_q	Elasticity of substitution between home and foreign goods	2
τ	Iceberg cost	varies
$\omega_1, \omega_2, \omega_3$	Home bias	[.64, .64, .89]
$\theta_1, \theta_2, \theta_3$	Cost share of consumption goods	[.03, .15, .82]
β	Discount factor	.96
η	Preference parameter	.36
ζ	Risk aversion	2
ρ	Persistent parameter of productivity shock	.89

[12] 14 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Korea, Netherlands, Norway, Spain, and Sweden.

I calibrate α_m, the parameter of intermediate goods share as 50%, which is a little bit higher than the average cost share of intermediate goods in U.S. 47%[13] but convenient for calculation. In the remaining value-added part (50%), I assume labor accounts for 64%, capital 36% according to the literature. Therefore, $\alpha_l = 50\% * 64\% = 32\%$, $\alpha_k = 50\% * 36\% = 18\%$. I calibrate the annual depreciation rate as $.1 = .025 * 4$, where .025 is a common quarterly depreciation rate in the literature, like in Backus, Kehoe, and Kydland (1995, [7]).

I calibrate the input-output matrix from U.S. 2002 input-output make-use matrix constructed by Bureau of Economic Analysis. The original input-output matrix at sectoral level contains 15 sectors and I aggregate them down to "three big". The input-output matrix in table 2 guarantees that cost shares of inputs in the steady state are consistent with data.

The elasticity of substitution between home and foreign goods ($\sigma_q = 2$) is greater than that between different sectors ($\sigma_m = .5$). Iceberg cost varies from 0 to 20%[14]. To calibrate the home bias parameter, I use U.S. 1997, 2002 input-output matrix to estimate the ratio of imported goods in total outputs for three sectors. The average import ratio for the first two sectors are .17 and .01 for service[15]. I set up the value of $\omega_1, \omega_2, \omega_3$ such that the shares of imported goods in steady state are consistent with data estimates.

As for preference parameters, I calibrate annual discount rate as 4%. I set the value of $\theta_1, \theta_2, \theta_3$ such that in steady state, the size of gross production in three sectors matches the average sector size in U.S.: [.10, .25, .65][16]. The value of utility parameter η guarantees the steady state of labor supply to be .329, which is consistent with U.S. data. I calibrate the risk aversion parameter ζ as 2 from the literature.

I estimate productivity shock process by running an AR(1) regression on the Solow residual vector $\hat{z} \equiv [\hat{z}^1, \hat{z}^2, \hat{z}^3, \hat{z}^{*1}, \hat{z}^{*2}, \hat{z}^{*3}]$, where $\hat{z}$ is logged and de-trended. I estimate the AR(1) process by Seemingly Unrelated Regression (SUR) method, restricting AR(1) coefficients for each element of $\hat{z}$ identical and no spill-over effect:

$$\hat{z}_t = A\hat{z}_{t-1} + \epsilon_t,$$

where $A = \rho * I$, ρ is the persistence parameter, I is an identity matrix, ϵ_t is i.i.d. normally distributed random vector with a symmetric variance-covariance matrix Σ. The results of esti-

[13]Data span: 1988-2004.
[14]Anderson and van Wincoop (2004, [4]) estimate the transportation cost for international trade is around 21%
[15]I obtained very similar results when I calibrated import ratios in the first two sectors as different numbers.
[16]Data resource: OECD STAN database

mation are

$$\rho = .89(.04);$$

and

$$\Sigma = 10^{-4} \times \begin{pmatrix} 1.09 & .27 & .27 & 0 & .09 & .19 \\ .27 & 1.55 & .08 & .09 & .88 & 0 \\ .26 & .08 & .46 & .19 & 0 & .36 \\ 0 & .09 & .19 & 1.09 & .27 & .26 \\ .09 & .88 & 0 & .27 & 1.55 & .08 \\ .19 & 0 & .36 & .26 & .08 & .46 \end{pmatrix}.$$

I make the covariance Σ symmetric using the method in Backus, Kehoe, and Kydland (1995, [7]): the standard deviation of the innovation is set to the average in U.S. and the rest of the world; the covariance is the product of correlation and revised standard deviations.

4.2 Quantitative analysis: the benchmark model

I simulate the benchmark model by the following procedure. I randomly draw an iceberg cost τ from a uniform distribution $[0, 0.2]$, draw a 67 period productivity innovation process from a normal distribution $\epsilon\ N(0, \Sigma)$, feed them into the model, and discard the initial 50 observations. I use the left 17 observations to compute trade intensity and output correlation because the time span in the data set is 17 years, from 1988 to 2004. I repeat this procedure 50 times and run regressions like in table 1.

Table 3 shows the main result: regressions based on simulation of the benchmark model are consistent with these based on data. In a single regression of real output correlation on trade intensity (the first row in (1), table 3), the model predicts a lighter lower slope than data estimation: .080 compared with .095 (the slope obtained by OLS regression) and .107 (the slope obtained by IR regression) in table 1. After controlling TFP correlation (the second row in (1), table 3), the benchmark model accounts for 100% the slope of trade intensity on output correlations. The slope of trade intensity on output correlation in the simulation regression is .050, between the OLS estimation .049 and IV estimation .067 in table 1.

Table 3: Regression from simulation

	(1)	Benchmark	(2)	No intermediate	(3)	V. production with IO	(4)	Vertical production
log(trade intensity)	.09*** (.02)	.07*** (.01)	.01 (.01)	.01 (.01)	.03*** (.01)	.02** (.01)	-.01 (.01)	-.02** (.01)
TFP correlation		.48*** (.06)		.17*** (.03)		.46*** (.06)		.75*** (.02)
R^2	.29	.69	.01	.42	.02	.55	.02	.94

*Note: regressand is bilateral correlation of real GDPs, HP filtered. Standard deviation recorded in parentheses. Intercepts not recorded. *** marks significance at 1% level.Sample size = 50. Data resource: model simulation.*

Table 4: Simulation result: volatility

	std (%)		rela.	std		
	y	c	i	l	rim	rex
U.S.data	1.48	.86	3.71	.68	3.95	3.75
(1)Benchmark	1.48	.49	2.28	.36	1.03	1.03
(2)No intermediate	.87	.48	3.22	.53	.86	.93
(3)Vertical production with IO	1.42	.48	2.20	.30	.91	.93
(4)Vertical production	.92	.48	3.22	.53	.86	.93

Note: U.S. data: 1988-2004, OECD Economic outlook. The rest of world (14 countries): Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Korea, Netherlands, Norway, Spain, Sweden.

Table 5: Simulation result: correlations with real output

	c	i	l	rim	rex
U.S.data	.89	.92	.92	.94	.69
(1)Benchmark	.87	.97	.96	.63	.96
(2)No intermediate	.59	.93	.86	.91	.99
(3)Vertical production with IO	.91	.96	.94	.70	.97
(4)Vertical production	.59	.93	.86	.91	.99

Note: see table 4.

Table 6: Simulation result: international correlations

	y	c	i	l
Data	.65	.55	.30	.28
(1)Benchmark	.59	.92	.84	.64
(2)No intermediate goods	.88	.17	.97	.99
(3)Vertical production with IO	.61	.92	.94	.96
(4)Vertical production	.88	.17	.97	.99

Note: see table 4.

4.3 Compare with other models

I construct three more models and compare them with the benchmark model, in order to detect what mechanism works. Model 2 in table 3 shuts down the channel of intermediate goods in the benchmark model by assuming the cost share of intermediate input in production α_m is zero. Model 3 nests input-output matrix into Burstein-Kurz-Tesar model. Model 4 is the Burstein-Kurz-Tesar model. It models vertically integrated composite assembled with a relatively low elasticity of substitution between home and foreign intermediate inputs. I choose Burstein-Kurz-Tesar model because it clearly models "in-line" production. By the comparison below, I show that only "roundabout", not "in-line" production structure can generate multiplier effect. Appendix B explains more detail about model 3 and 4.

In summary, there are two groups of models: model 1 and 2; model 3 and 4. The first model in each group has the feature of input-output matrix, which the second model does not. If the multiplier effect generated by the interaction of intermediate goods is the key mechanism to explain higher trade intensity tend to make country outputs synchronized, models with input-output matrix in every group: model 1 and mode 3 should should perform better. Quantitative exercises verify this conjecture in table 3. Below I explain model comparison in detail.

Model 2 is a standard two-country real business cycle model except that there are multiple sectors in one country. I do not simplify it to one-sector model just for the convenience of comparison with the benchmark model. Deleting the intermediate good structure in model 2 leads to ambiguous relationship between trade intensity and output correlation, as shown in table 3. The coefficients of trade intensity in the regression are not significant, either controlling or not TFP correlation. In summary, after shutting down the channel of multiplier effect, the complementarity effect in the standard model is not enough to dominate the motive of producing goods in the country with favorable productivity shocks.

Comparing model 3 and 4 again proves that it is the multiplier effect which can explain the positive relationship between trade and output correlation. Following Burstein, Kurz, and Tesar's modeling, incorporating vertical production adds some complementarity because the elasticity of substitution in vertical production is relatively low. But this addition is not enough to cancel out the motive producing goods in the country with favorable productivity shock, which tends to make outputs across countries move asynchronous. After nesting input-output matrix into Burstain-Kurze-Tesar model, model 3 reproduces positive relationship between bilateral trade and output correlation quite well.

To check other dimensions of these four models, I simulate them at 10% iceberg cost to calculate the first and second order moments. Results about moment statistics can be summarized as follows. (1) With input-output matrix, models require less volatile technology shocks to generate enough variance in real output. In table 4, the benchmark model quite successfully replicates the volatility of real GDP (1.33%) compared with U.S. data (1.48%), while the model with no intermediate good structure (model 2) only can explain have of real output variance in data (.70% vs. 1.48%). Comparing model 3 and model 4, again model 3 with input-output matrix generate higher variance in real output. (2) Models with input-output matrix generate lower relative volatilities of investment and labor with respect to real output. For example, the relative variance of investment with respect to real output is 2.28 in the benchmark model, below the data 3.71 and than in the model with no intermediate goods 3.25. (3) Models with input-output matrix match correlation between real GDP and other aggregate variables with data, as documented in table 5. (4) Considering international correlations in table 6, models with input-output matrix can not remedy the quantity anomaly proposed by Backus, Kehoe and Kydland ([7] ,1995). As the benchmark model assumes complete assets market, consumers can fully share risks such that consumptions are more correlated across countries than outputs.

4.4 Intuition

The variation of multiplier effect with different trade intensities is the key mechanism in the benchmark model (model 1). The standard model does not have this characteristics.

The multiplier effect varies with trade intensities in the benchmark model, which results in higher output correlations under higher trade intensities. When trade barrier is high, one industry acquires few imported goods from abroad in every round and leads to a low multiplier effect in the foreign country. On the other hand, if the trade barrier is low, every industry needs more intermediate goods shipping from abroad in each round, which results in a high multiplier effect. Foreign outputs increase more persistently and output correlation across countries is higher. Figure 2 illustrates the different intensity of multiplier effect. The solid lines show responses of home and foreign real outputs after one-percent shock to the level of technology in the home country, assuming the iceberg cost $\tau = .2$. The dotted lines show responses of the same variables while assuming the iceberg cost $\tau = .01$. Comparing these two scenarios, low iceberg cost($\tau = .01$) cleans up the blockage of trade across countries; foreign output bounce up to a higher level and is persistently greater than the output response under high iceberg cost ($\tau = .2$). In other words, the technology shock in the home country drives foreign output in a high and persistent way under low iceberg cost and thus generates higher output correlation in the new model.

In contrast, there is no multiplier effect in the standard model (model 2). Furthermore, there is no other factor making output correlation change with the variation of trade intensities. Fig 3 shows that the responses of foreign output to one percent home technology shock under iceberg cost $\tau = .2$ (solid line) and $\tau = .01$ (dotted line) are almost unchanged. Therefore, output correlations across countries are almost constant under different trade intensities. This is why the standard model generates lower positive slope of output correlations on trade intensities than data estimation.

In summary, the benchmark model (model 1) can generate higher output correlations under higher trade intensity, because more trade leads to higher multiplier effect in the foreign country. The larger and more persistent output increase in the foreign country makes it more correlated with the output in the home country under higher trade intensities. The standard model (model 2) ends up with almost no change in output correlations under different trade intensities, as a single round of requirement for the foreign goods can not yield different magnitude of output increase in the foreign country.

5 Conclusion and future plan

This paper proposes a new channel to explain the international transmission of business cycle through bilateral trade. A positive productivity shock in one sector can spread to the whole economy by requiring intermediate goods from other sectors, and other sectors require more

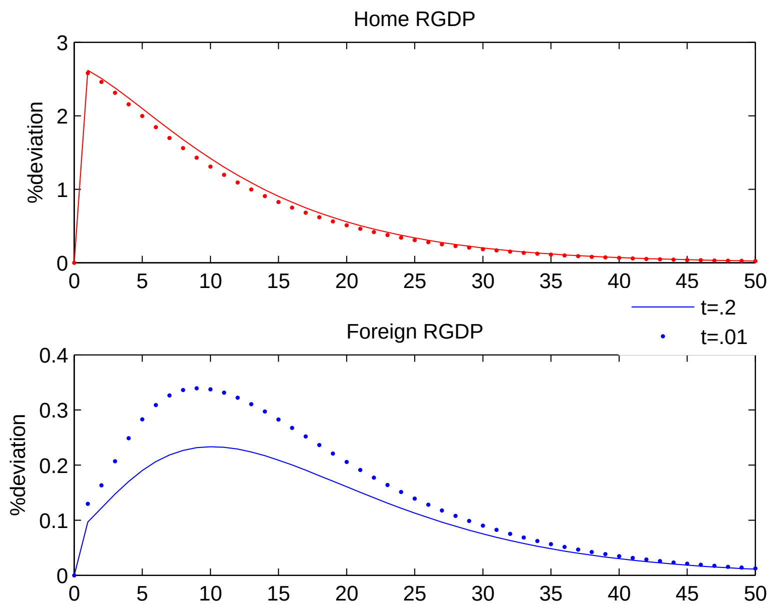

Figure 2: Impulse-response functions in the new model

The figure shows the responses of real outputs in the home and foreign country in percentage deviations from the steady state, as a result of one percent shock to the level of technology (in all sectors) in the home country. Each period refers to one year.

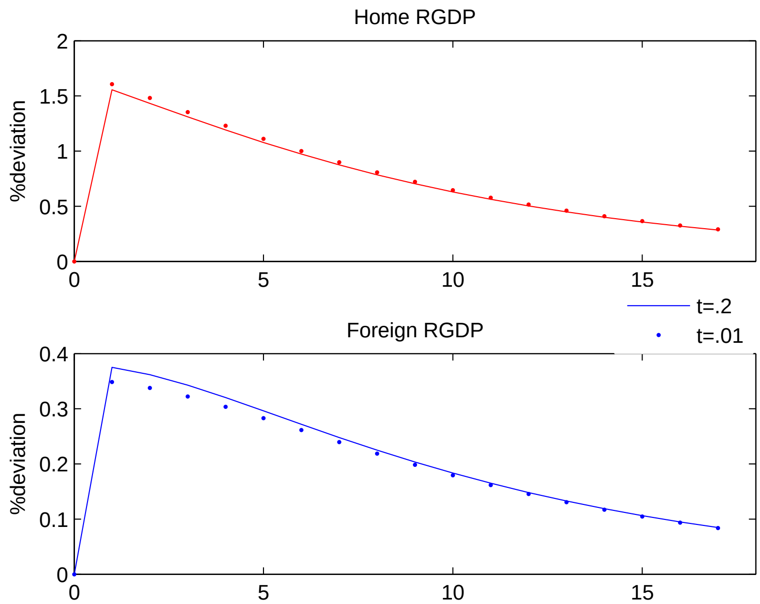

Figure 3: Impulse-response functions in the standard model

The figure shows the responses of real outputs in the home and foreign country in percentage deviations from the steady state, as a result of one percent shock to the level of technology (in all sectors) in the home country. Each period refers to one year.

inputs in further rounds. Multiplier effects from the interaction in the input-output table enlarge the complementarity effect. Simulation results from the new model are consistent with data estimation.

References

[1] Steve, Ambler, Emanuela Cardia, Christian Zimmermann, 2002. "International Transmission of the Business Cycle in a Multi-sector Model." European Economic Review, vol. 46, pp. 273-200.

[2] Alessandria, George and Joseph Koboski, 2004. "Violating Purchaing Power Parity." Federal Reserve Bank of Philadelphia working paper.

[3] Arkolakis, Costas and Ananth Ramanarayanan, 2008. "Vertical Specialization and International Business Cycle Synchronization." Federal Reserval Bank of Dallas working paper no. 21.

[4] Anderson, James E., Eric van Wincoop, 2004. "Thrde Costs". Journal of Economic Literature. vol.42, no.3, pp.691-751.

[5] Basu, Susanto, 1995. "Intermediate Goods and business Cycles: Implications for Productivity and Welfare." American Economic Review, vol.85, no.3, pp.512-531.

[6] Baxter, Marianne and Michael A. Kouparitsas, 2005. "Determinants of Business Cycles Movement: A Robust Analysis." Jounal of Monetary Economics, vol. 52, no. 1, pp. 113-57.

[7] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, 1995. "International business Cycles: Theory vs. Evidence." in Thomas F.Cooley, ed., Frontiers of Business Cycle Research, Princeton University Press.

[8] Burstein, Ariel, Christopher Kurz,and Linda Tesar, 2008. "Trade, Production Sharing, and the Intermational Transmission of Business Cycles."

[9] Calderon, Cesar, Alberto Chong, and Ernesto Stein, 2007. "Trade intensity and Business Cycle Synchronization: Are Developing Countries any Different?" Journal of International Economics, vol. 71, no. 1, pp. 2-21.

[10] Clark, Todd E. and Eric van Wincoop, 2001. "Borders and Business Cycles." Journal of International Economics, vol. 55, no. 1, pp. 59-85.

[11] Deardorff, Alan V., 1998. Determinants of Bilateral Trade: Does Gravity Work in a Neoclassical World? In *The Regionalization of the World Economy*, Jeffery Frankel eds.(University of Chicago Press). pp.7-22.

[12] Drozd, Lukasz A., Jaromir B.Nosal, 2008. "Long-Run Price Elasticity of Trade and the Trade-Comovement Puzzle." working paper.

[13] Frankel, Jeffrey A. and Andrew K. Rose, 1998. "The Endogeneity of the Optimum Currency Area Criteria." Economic Journal, vol. 108, July, pp 1009-25.

[14] di Giovanni, Julian, Andrei A. Levchenko, 2008. "Putting the Parts Together: Trade, Vertical Linkage and Business Cycle Comovement." working paper.

[15] Heathcote, Johnathan, and Fabrizio Perri, 2002. "Financial Autarky and International Business Cycles." Journal of Monetary Economics, vol.49, no.3, pp 601-27.

[16] Kose, M.Ayhan and Kei-Mu Yi, 2001. "International Trade and Business Cycles: Is Vertical Intergration the Missing Link." American Economic Review, vol. 91, no.2, 371-75.

[17] Kose, M.Ayhan and Kei-Mu Yi, 2006."Can the Standard International business Cycle model Explain the Relation Between Trade and Comovement?" Journal of International Economics, vol. 68, no.2, pp. 267-95.

[18] Ramanarayanan, Ananth, 2009. "Ties That Bind: Bilateral Trade's Role in Synchronizing Business Cycles." Economic Letter, vol.4, no.1, Federal Reserve Bank of Dallas.

[19] Yi, Kei-Mu, 2003. "Can Vertical Specialization Explain the Growth of World Trade?" Journal of Political Economy, vol. 111, no. 1, pp. 52-102.

Appendices

A The benchmark model

A.1 Equilibrium conditions

Demand equations

$$p_{d,t}^{h} = p_{q,t}^{h} \cdot \frac{\partial q_{h,t}}{\partial d_t^h}, \quad h = 1, 2, 3 \tag{14}$$

$$p_{f,t}^{h} = p_{q,t}^{h} \cdot \frac{\partial q_{h,t}}{\partial f_t^h}, \quad h = 1, 2, 3 \tag{15}$$

$$p_{d,t}^{*h} = p_{q,t}^{*h} \cdot \frac{\partial q_{h,t}^*}{\partial d_t^{*h}}, \quad h = 1, 2, 3 \tag{16}$$

$$p_{f,t}^{*h} = p_{q,t}^{*h} \cdot \frac{\partial q_{h,t}^*}{\partial f_t^{*h}}, \quad h = 1, 2, 3 \tag{17}$$

Euler equations and consumption-leisure trade off

$$U_c(c_t, l_t) = \beta E_t[U_c(c_{t+1}, l_{t+1}) \cdot (1 - \delta + r_{t+1})] \tag{18}$$

$$U_{c^*}(c_t^*, l_t^*) = \beta E_t[U_{c^*}(c_{t+1}^*, l_{t+1}^*) \cdot (1 - \delta + r_{t+1}^*)] \tag{19}$$

$$U_c(c_t, l_t) v(s^t, s_{t+1}) = prob(s_{t+1}) \beta U_c(c_{t+1}(s_{t+1}), l_{t+1}(s_{t+1})) \quad \text{for all} s_{t+1} \tag{20}$$

$$U_{c^*}(c_t^*, l_t^*) \frac{x(s^t, s_{t+1})}{x_t} v(s^t, s_{t+1}) = prob(s_{t+1}) \beta U_{c^*}(c_{t+1}^*(s_{t+1}), l_{t+1}^*(s_{t+1})) \quad \text{for all} s_{t+1} \tag{21}$$

$$-\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = w_t \tag{22}$$

$$-\frac{U_{l^*}(c_t^*, l_t^*)}{U_{c^*}(c_t^*, l_t^*)} = w_t^* \tag{23}$$

Risk sharing

I assume two countries are ex-ante symmetric:

$$\frac{U_{c^*}(c_t^*, l_t^*)}{U_c(c_t, l_t)} = x_t \tag{24}$$

Law of one price

$$p_{d,t} = x_t \cdot (1-\tau) \cdot p^{*h}_{d,t}, \quad h = 1,2,3 \tag{25}$$

$$p_{f,t} = x_t \cdot (1-\tau)^{-1} \cdot p^{*h}_{f,t}, \quad h = 1,2,3 \tag{26}$$

Factor prices

$$p^h_{d,t} \cdot \alpha_k \frac{y^h_t}{k^h_t} = r_t, \quad h = 1,2,3 \tag{27}$$

$$p^h_{d,t} \cdot \alpha_l \frac{y^h_t}{l^h_t} = w_t, \quad h = 1,2,3 \tag{28}$$

$$p^{*h}_{f,t} \cdot \alpha_k \frac{y^{*h}_t}{k^{*h}_t} = r^*_t, \quad h = 1,2,3 \tag{29}$$

$$p^{*h}_{f,t} \cdot \alpha_l \frac{y^{*h}_t}{l^{*h}_t} = w^*_t, \quad h = 1,2,3 \tag{30}$$

Intermediate goods

$$p^h_{m,t} = p^h_{d,t} \cdot \alpha_m \frac{y^h_t}{m^h_t}, \quad h = 1,2,3 \tag{31}$$

$$p^{*h}_{m,t} = p^{*h}_{f,t} \cdot \alpha_m \frac{y^{*h}_t}{m^{*h}_t}, \quad h = 1,2,3 \tag{32}$$

$$q^j_{i,t} = \gamma_{ij}^{\sigma_m} \cdot (\frac{p^i_{q,t}}{p^j_{m,t}})^{-\sigma_m} \cdot m^j_t, \quad i,j = 1,2,3 \tag{33}$$

$$q^{*j}_{i,t} = \gamma_{ij}^{\sigma_m} \cdot (\frac{p^{*i}_{q,t}}{p^{*j}_{m,t}})^{-\sigma_m} \cdot m^{*j}_t, \quad i,j = 1,2,3 \tag{34}$$

After normalizing price of consumption and investment as one,

$$p^h_{q,t} = \frac{\theta_h(c_t + i_t)}{q^0_{h,t}} \quad h = 1,2,3 \tag{35}$$

$$p^{*h}_{q,t} = \frac{\theta_h(c^*_t + i^*_t)}{q^{*0}_{h,t}} \quad h = 1,2,3 \tag{36}$$

Feasibility and market clearing conditions

$$y_t^h = z_t^h (k_t^h)^{\alpha_k} (l_t^h)^{\alpha_l} (m_t^h)^{\alpha_m}, \quad h = 1,2,3 \tag{37}$$

$$y_t^{*h} = z_t^{*h} (k_t^{*h})^{\alpha_k} (l_t^{*h})^{\alpha_l} (m_t^{*h})^{\alpha_m}, \quad h = 1,2,3 \tag{38}$$

$$d_t^h + \frac{d_t^{*h}}{1-\tau} = y_t^h, \quad h = 1,2,3 \tag{39}$$

$$f_t^{*h} + \frac{f_t^h}{1-\tau} = y_t^{*h}, \quad h = 1,2,3 \tag{40}$$

$$q_{h,t} = (\omega_h (d_t^h)^{\frac{\sigma_q - 1}{\sigma_q}} + (1-\omega_h)(f_t^h)^{\frac{\sigma_q - 1}{\sigma_q}})^{\frac{\sigma_q}{\sigma_q - 1}}, \quad h = 1,2,3 \tag{41}$$

$$q_{h,t}^* = (\omega_h (f_t^{*h})^{\frac{\sigma_q - 1}{\sigma_q}} + (1-\omega_h)(d_t^{*h})^{\frac{\sigma_q - 1}{\sigma_q}})^{\frac{\sigma_q}{\sigma_q - 1}}, \quad h = 1,2,3 \tag{42}$$

$$m_t^h = (\gamma_{1h}(q_{1,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{2h}(q_{2,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{3h}(q_{3,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}})^{\frac{\sigma_m}{\sigma_m - 1}}, h = 1,2,3 \tag{43}$$

$$m_t^{*h} = (\gamma_{1h}(q_{1,t}^{*h})^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{2h}(q_{2,t}^{*h})^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{3h}(q_{3,t}^{*h})^{\frac{\sigma_m - 1}{\sigma_m}})^{\frac{\sigma_m}{\sigma_m - 1}}, h = 1,2,3 \tag{44}$$

$$c_t + i_t = (q_{1,t}^0)^{\theta_1}(q_{2,t}^0)^{\theta_2}(q_{3,t}^0)^{\theta_3} \tag{45}$$

$$c_t^* + i_t^* = (q_{1,t}^{*0})^{\theta_1}(q_{2,t}^{*0})^{\theta_2}(q_{3,t}^{*0})^{\theta_3} \tag{46}$$

$$\sum_{j=1}^{3} q_{h,t}^j + q_{h,t}^0 = q_{h,t}, h = 1,2,3 \tag{47}$$

$$\sum_{j=1}^{3} q_{h,t}^{*j} + q_{h,t}^{*0} = q_{h,t}^*, h = 1,2,3 \tag{48}$$

$$k_t = \sum_{h=1}^{3} k_t^h \tag{49}$$

$$k_t^* = \sum_{h=1}^{3} k_t^{*h} \tag{50}$$

$$k_{t+1} = (1-\delta)k_t + i_t \tag{51}$$

$$k_{t+1}^* = (1-\delta)k_t^* + i_t^* \tag{52}$$

$$l_t = \sum_{h=1}^{3} l_t^h \tag{53}$$

$$l_t^* = \sum_{h=1}^{3} l_t^{h*} \tag{54}$$

$$b_t + x_t b_t^* = 0 \tag{55}$$

Equation (14) to (55) can solve prices $\{p_{d,t}^h\}, \{p_{d,t}^{*h}\}, \{p_{f,t}^h\}, \{p_{f,t}^{*h}\}, \{p_{q,t}^h\}, \{p_{q,t}^{*h}\}, \{p_{m,t}^h\}, \{p_{m,t}^{*h}\}$, $\{w_t\}, \{w_t^*\}, \{r_t\}, \{r_t^*\}, \{x_t\}, \{v(s^t, s_{t+1})\}, h = 1, 2, 3$
and allocations $\{y_t^h\}, \{y_t^{*h}\}, \{d_t^h\}, \{d_t^{*h}\}, \{f_t^{*h}\}, \{f_t^h\}, \{c_t\}, \{c_t^*\}, \{i_t\}, \{i_t^*\}, \{k_t^h\}, \{k_t^{*h}\}, \{k_t\}, \{k_t^*\}$ $\{l_t^h\}, \{l_t^{*h}\}, \{l_t\}, \{l_t^*\}, \{q_{h,t}\}, \{q_{h,t}^*\}, \{q_{h,t}^j\}, \{q_{h,t}^{*j}\}, \{q_{h,t}^0\}, \{q_{h,t}^{*0}\}, \{m_t^h\}, \{m_t^{*h}\}, \{b_t\}, \{b_t^*\}, h, j = 1, 2, 3$.

A.2 Solving steady state of the benchmark model

I focus on the case of home country because of the symmetry between two countries.

In the steady state, under symmetry, $p_f^h = \frac{p_d^h}{1-\tau}, h = 1, 2, 3$. Solving equation (14) and (15) gives

$$p_q^h = (\omega_h^{\sigma_q}(p_d^h)^{1-\sigma_q} + (1-\omega_h)^{\sigma_q}(p_f^h)^{1-\sigma_q})^{\frac{1}{1-\sigma_q}}, \quad h = 1, 2, 3. \tag{56}$$

With $p_f^h = \frac{p_d^h}{1-\tau}$, the price of the composite goods in sector h is proportional to its domestic price in the steady state,

$$p_q^h = W_h p_d^h, \tag{57}$$

where $W_h = (\omega_h^{\sigma_q} + (1-\omega_h)^{\sigma_q}(1-\tau)^{\sigma_q - 1})^{\frac{1}{1-\sigma_q}}$. Furthermore, the price of intermediate goods can be simplified to a combination of domestic good prices:

$$p_m^h = (\gamma_{1h}^{\sigma_m}(p_q^1)^{1-\sigma_m} + \gamma_{2h}^{\sigma_m}(p_q^2)^{1-\sigma_m} + \gamma_{3h}^{\sigma_m}(p_q^3)^{1-\sigma_m})^{\frac{1}{1-\sigma_m}}, \quad h = 1, 2, 3. \tag{58}$$

Because firms are perfectly competitive, the price of goods in sector h equals its marginal cost:

$$p_d^h = \frac{1}{\alpha_k^{\alpha_k}\alpha_l^{\alpha_l}\alpha_m^{\alpha_m}} w^{\alpha_l} r^{\alpha_k} (p_m^h)^{\alpha_m}. \tag{59}$$

Then the price ratio of two sectors depends on the price ratio of intermediate goods:

$$\frac{p_d^1}{p_d^3} = (\frac{p_m^1}{p_m^3})^{\alpha_m} \tag{60}$$
$$\frac{p_d^2}{p_d^3} = (\frac{p_m^2}{p_m^3})^{\alpha_m} \tag{61}$$

As pointed out in (58), intermediate good prices are the combination of domestic good prices. Thus (57), (60) and (61) pin down the relative price $\frac{p_d^1}{p_d^3}, \frac{p_d^2}{p_d^3}$. To solve the level of domestic good prices, I normalize the price of consumption good as 1, and from (45)

$$\frac{1}{\theta_1^{\theta_1}\theta_2^{\theta_2}\theta_3^{\theta_3}}(p_q^1)^{\theta_1}(p_q^2)^{\theta_2}(p_q^3)^{\theta_3} = 1. \tag{62}$$

With relative price equation (60) and (61), one can solve domestic good price $p_d^h, h = 1, 2, 3$, then $p_q^h, p_m^h, h = 1, 2, 3$.

The relative size of each sector depends on how the good in each sector can be used as intermediate or final goods. Using the symmetry condition and substituting demand function of composite goods (33) and (35) into the market clearing condition (47) can solve out the relative size $[G_1, G_2, G_3] \equiv [\frac{p_d^1 y_d^1}{\sum_{h=1}^3 p_d^h y_d^h}, \frac{p_d^2 y_d^2}{\sum_{h=1}^3 p_d^h y_d^h}, \frac{p_d^3 y_d^3}{\sum_{h=1}^3 p_d^h y_d^h}]$.

Given labor supply in the steady state $l = .329$, labor demand in sector h is

$$l^h = G_h \cdot l, \quad h = 1, 2, 3 \tag{63}$$

From Euler equation (18), the capital rent in steady state is

$$r = \frac{1}{\beta} - (1 - \delta). \tag{64}$$

With labor demand (63) and sector size, one can solve aggregate capital k and capital demand $k^h, h = 1, 2, 3$. The investment in steady state is $i = \delta k$.

It is ready to solve all other variables. consumption can be solved from household demand equation (35) and aggregation equation for consumption and investment (45), given investment i and sector size G_h. Production in each sector is solved from demand for capital equation (27):

$$y^h = G_h \frac{rk}{\alpha_k}\frac{1}{p_d^h}, \quad h = 1, 2, 3. \tag{65}$$

Wage can be solved from its relative demand with capital:

$$w = \frac{\alpha_l}{\alpha_k}\frac{k}{r}\frac{1}{l}. \tag{66}$$

Intermediate goods in each sector m^h is solved from its demand function (31)

$$m^h = \frac{\alpha_m p_d^h y^h}{p_m^h}, \quad h = 1, 2, 3; \tag{67}$$

and q_h^s from the demand function (33)

$$q_h^s = \gamma_{hs}^{\sigma_m} (\frac{p_q^h}{p_m^s})^{-\sigma_m} m^s, \quad h, s = 1, 2, 3. \tag{68}$$

B The model with vertical production

B.1 Burstein, Kurz, and Tesar model

Burstain-Kurz-Tesar model is an "in-line" production model as production proceeds synchronically: after producing out intermediate goods, firms in the final goods sector use the inputs to produce consumption goods. Firms can not serve both intermediate and final good market. I do not model nontraded goods as in the original model. This change simplifies the model and does not change the main implication. Actually, Burstein, Kurz, and Tesar consider the output correlation in the traded good across countries in their paper, not the correlation of aggregate output.

Burstein, Kurz, and Tesar model vertically integrated composite assembled with a relatively low elasticity of substitution between home and foreign intermediate inputs, compared with horizontally differentiated goods. In the home country, there are two kinds of intermediate composites: the horizontally differentiated goods and vertically integrated goods. The horizontally differentiated goods x_t^h combines home and imported intermediate goods d_t^h, f_t^h by

$$X_t^h = (\Omega_h (d_t^h)^{\frac{\sigma_x}{\sigma_x - 1}} + (1 - \Omega_h)(f_t^h)^{\frac{\sigma_x}{\sigma_x - 1}})^{\frac{\sigma_x - 1}{\sigma_x}}, \quad h = 1, 2, 3. \tag{69}$$

The vertically differentiated goods V_t^h again are a composite of home and imported inputs v_t^h, v_t^{*h}, but inputs are assembled with a relatively low elasticity of substitution $\sigma_v < \sigma_x$:

$$V_t^h = (\lambda_h (v_t^h)^{\frac{\sigma_v}{\sigma_v - 1}} + (1 - \lambda_h)(v_t^{*h})^{\frac{\sigma_v}{\sigma_v - 1}})^{\frac{\sigma_v - 1}{\sigma_v}}, \quad h = 1, 2, 3. \tag{70}$$

Now the composite q_t^h in the home country is no longer (41) but

$$q_t^h = (X_t^h)^{\nu}(V_t^h)^{1-\nu}. \tag{71}$$

All composites are used as consumption and investment goods:

$$c_t + i_t = (q_t^1)^{\theta_1}(q_t^2)^{\theta_2}(q_t^3)^{\theta_3} \tag{72}$$

The remaining part is similar to the benchmark model.

B.2 Nesting input-output structure into the Burstein, Kurz, and Tesar model

This model is identical with the model above except the production process. Specifically, a representative competitive firm in sector $h = 1, 2, 3$ in the home country produces goods according to

$$y_t^h = z_t^h(l_t^h)^{\alpha_l}(k_t^h)^{\alpha_k}(m_t^h)^{\alpha_m}, h = 1, 2, 3 \tag{73}$$

where the intermediate goods are

$$m_t^h = (\gamma_{1h}(q_{1,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{2h}(q_{2,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}} + \gamma_{3h}(q_{3,t}^h)^{\frac{\sigma_m - 1}{\sigma_m}})^{\frac{\sigma_m}{\sigma_m - 1}}, h = 1, 2, 3. \tag{74}$$

The remaining of composite goods are used as consumption and investment:

$$c_t + i_t = (q_t^1 - \sum_{h=1}^{3} q_{1,t}^h)^{\theta_1}(q_t^2 - \sum_{h=1}^{3} q_{2,t}^h)^{\theta_2}(q_t^3 - \sum_{h=1}^{3} q_{3,t}^h)^{\theta_3}. \tag{75}$$